Instituto de Matemática e Estatística
Universidade Federal Fluminense

# Seminário de Sistemas Dinâmicos da UFF

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## Existence of common zeros for commuting vector fields on 3-manifolds


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IMPA


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**Hora:** 14h30.
**Local:** Sala 207, Bloco G, Campus Gragoatá.

**Resumo**


The Poincaré-Hopf Theorem asserts that topological information of a manifold can be obtained by looking at zeros of vector fields. Conversely, the topology can force the existence of zeros. It is natural then to search for similar relations between the topology of the ambient manifold and fixed points of more general group actions. In this direction, Elon Lima showed in the late 60's that on every closed surface with non-vanishing Euler characteristic every Rn-action has a fixed point, which is the same as to say that every set of n-pairwise commuting vector fields on the surface has a common zero.

In higher dimensions, the only result about existence of fixed points is due to Christian Bonatti, in 92. He proved that on real analytic manifold $M$, with dim $M$ less or equal than 4, every pair $X$ and $Y$ of analytic commuting vector fields has a common zero inside any compact region $U$ where the Poincaré-Hopf index $Ind(X, U)$ of $X$ at $U$ is non-vanishing.

In this talk I shall explain a recent result, obtained in collaboration with Christian Bonatti, where we prove existence of common zeros for $C^1$ commuting vector fields $X$ and $Y$ on any compact region $U$ of a 3-manifold $M$, if $Ind(X, U)$ is not zero and with the additional assumption that the set of points where $X$ and $Y$ are collinear is contained in some closed surface embedded in $M$.